FIR

FIR FASE LINEAR

* DEFINIÇÃO DE ATRASO DE GRUPO $\tau = \frac{d \measuredangle H(\omega)}{d\omega}$

* FASE LINEAR É IMPORTANTE

$$H(e^{j\omega}) = B(\omega).e^{-j\omega\tau + j\phi} \quad \begin{cases} B(\omega) - \text{REAL} \\ \tau \in \phi - \text{CONSTANTES} \end{cases}$$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} B(\omega).e^{-j\omega\tau + j\phi}.e^{j\omega n} d\omega$$

$$= \frac{e^{j\phi}}{2\pi}\int_{-\pi}^{\pi} B(\omega).e^{j\omega(n-\tau)} d\omega$$

VAMOS FAZER $\tau = \frac{k}{2}$, $k \in \mathbb{Z}$, LOGO

$$h(2\tau - n) = \frac{e^{j\phi}}{2\pi}\int_{-\pi}^{\pi} B(\omega).e^{j\omega(2\tau - n - \tau)} d\omega$$

$$= \frac{e^{j\phi}}{2\pi}\int_{-\pi}^{\pi} B(\omega).e^{j\omega(\tau - n)} d\omega$$

COMO $B(\omega)$ É REAL

$$h^*(2\tau - n) = \frac{e^{-j\phi}}{2\pi}\int_{-\pi}^{\pi} B^*(\omega).e^{-j\omega(\tau - n)} d\omega$$

$$= \frac{e^{-j\phi}}{2\pi}\int_{-\pi}^{\pi} B(\omega).e^{j\omega(n-\tau)} d\omega$$

ENTÃO $h(m) = e^{2j\phi} h^*(2\tau - m)$

COMO QUEREMOS $h(m)$ CAUSAL E DURAÇÃO FINITA

$0 \le m \le M$ ENTÃO $\tau = \frac{M}{2}$, E

$$h(m) = e^{j2\phi} h^*(M - m)$$

SE QUISERMOS QUE $h(m)$ SEJA REAL $h(m) = h^*(m)$

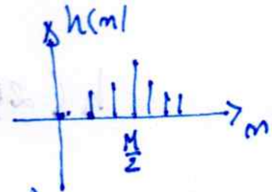
ENTÃO $e^{j2\phi}$ DEVE SER REAL, ENTÃO

$$\phi = \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

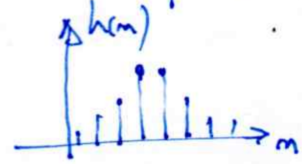
E $h(m) = (-1)^k h(M - m), \; k \in \mathbb{Z}$.

<u>OBS</u>: DEPENDENDO DOS VALORES DE k E M, $h(m)$ PODE SER SIMÉTRICA OU ANTI-SIMÉTRICA

1) $k = 0$, M PAR: $h(m) = h(M - m)$

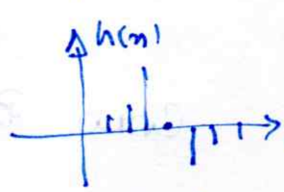


2) $k = 0$, M ÍMPAR: SIMÉTRICA

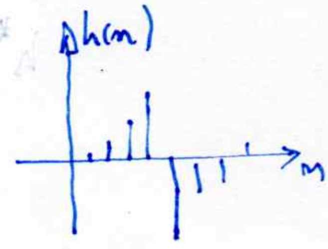


3) $k = 1$, M PAR: ANTI-SIMÉTRICA

$h(M/2) = 0$



4) $k = 1$ M ÍMPAR, ANTI-SIMÉTRICA



VEJA QUE $H(e^{jw}) = B(w).e^{-jw\frac{M}{2}+j\frac{k\pi}{2}}$

TIPO I : $K=0$, M PAR

$$H(z) = \sum_{m=0}^{\frac{M}{2}-1} h(m).z^{-m} + h\left(\frac{M}{2}\right)z^{-\frac{M}{2}} + \sum_{m=\frac{M}{2}+1}^{M} h(m).z^{-m}$$

$$= \sum_{m=0}^{\frac{M}{2}-1} h(m)\left[z^{-m} + z^{-(M-m)}\right] + h\left(\frac{M}{2}\right)z^{-M/2}$$

OBS: RELEMBRAR A RELAÇÃO TRANSFORMADA Z E FOURIER.

AVALIANDO NO CÍRCULO UNITÁRIO $z \to e^{jw}$

$$H(e^{jw}) = \sum_{m=0}^{\frac{M}{2}-1} h(m)\left[e^{-jwm} + e^{-jw(M-m)}\right] + h\left(\frac{M}{2}\right)e^{-jw\frac{M}{2}}$$

$$= e^{-jw\frac{M}{2}}\left\{h\left(\frac{M}{2}\right) + \sum_{m=0}^{\frac{M}{2}-1} 2h(m)\cos\left[w\left(m-\frac{M}{2}\right)\right]\right\}$$

FAZENDO $\frac{M}{2} - m = m$ TEMOS

$$H(e^{jw}) = e^{-jw\frac{M}{2}}\left\{\sum_{m=0}^{\frac{M}{2}} a(m).\cos(wm)\right\}.$$

SENDO $a(0) = h\left(\frac{M}{2}\right)$ E $a(m) = 2h\left(\frac{M}{2} - m\right)$, $m = 1, \ldots, \frac{M}{2}$.

FIR-4

TIPO II: $H(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \sum_{m=1}^{\frac{M+1}{2}} b(m)\cdot \cos\left[\omega\left(m-1/2\right)\right]$

$$b(m) = 2h\left(\frac{M+1}{2}-m\right),\quad m = 1, 2, \dots, \frac{M+1}{2}.$$

OBS: PARA $\omega=\pi$ $H(e^{j\omega}) = 0 \Rightarrow$ NÃO ADEQUADOS PARA FILTROS PASSA-AL

TIPO III: $H(e^{j\omega}) = e^{-j\left(\omega\frac{M}{2}-\frac{\pi}{2}\right)} \sum_{m=1}^{\frac{M}{2}} c(m)\cdot \text{sen}(\omega m)$

$$c(m) = 2h\left(\frac{M}{2}-m\right),\quad m = 1, 2, \dots, \frac{M}{2}.$$

$H(e^{j\omega}) = 0$ PARA $\omega=0$ E $\omega=\pi \Rightarrow$ BOM PARA FILTRO PASSA-B

TIPO IV: $H(e^{j\omega}) = e^{-j\left(\omega\frac{M}{2}-\frac{\pi}{2}\right)} \sum_{m=1}^{\frac{M+1}{2}} d(m)\cdot \text{sen}\left[\omega\left(m-1/2\right)\right]$

$$d(m) = 2h\left(\frac{M+1}{2}-m\right),\quad m = 1, 2, \dots, \frac{M+1}{2}$$

OBS: $H(e^{j\omega}) = 0$ EM $\omega=0 \Rightarrow$ NÃO PODEM IMPLE MONTAR PASSA-BAIX